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GENERAL MOTORS CORPORATION

TECHNICAL REPORT

ON

**PLASMA FREQUENCY AND  
ELECTRON COLLISION  
FREQUENCY CHARTS  
FOR HYPERSONIC VEHICLE  
EQUILIBRIUM FLOW FIELDS IN AIR**

CONTRACT NUMBER DA-04-495-ORD-3567  
HYPERVELOCITY RANGE RESEARCH PROGRAM

DEFENSE RESEARCH LABORATORIES  
SANTA BARBARA, CALIFORNIA



AEROSPACE OPERATIONS DEPARTMENT



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**PLASMA FREQUENCY AND  
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FOR HYPERSONIC VEHICLE  
EQUILIBRIUM FLOW FIELDS IN AIR**

**PREPARED BY HENRY M. MUSAL, JR.**

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## ABSTRACT

The purpose of this report is to present charts of plasma frequency and electron collision frequency as functions of air density (altitude) and vehicle velocity for selected regions of equilibrium hypersonic flow fields, with a brief description of the method of derivation and the area of applicability.

The two particular regions of the flow field treated here are the stagnation region and the end-point of the isentropic (expansion controlled) part of the wake of a blunt body. The charts cover the altitude range from zero to 350,000 feet and the velocity range from 6,000 to 40,000 feet/second. \*

\* This research is a part of Project DEFENDER, Sponsored by The Advanced Research Projects Agency, Department of Defense.

## INTRODUCTION

At the present time, some of the properties of hypersonic vehicle flow fields in air can be theoretically predicted under certain assumptions and in particular parameter regimes. The two basic properties of greatest importance to the study of electromagnetic wave propagation in these flow fields are the plasma frequency and the electron collision frequency. Preliminary charts of these properties as functions of the aerodynamic variables have been prepared for selected parts of the flow field of a hypersonic blunt body. The regions of the flow field treated here are the stagnation region and the end-point of the isentropic (expansion controlled) part of the wake.

Using air density (altitude) and vehicle velocity as the aerodynamic variables, the maps cover the range from zero to 350,000 feet and from 6,000 to 40,000 feet/second, respectively. A brief description of these charts, the theoretical background upon which they are based, and the regime of applicability follow.

The charts presented here for the plasma frequency and electron collision frequency in hypersonic flow fields were derived on the basis of a number of assumptions that are not always met in aerodynamic situations of interest. That is, when the body is not blunt,

when chemical equilibrium is not maintained throughout the flow field, or when the flow is turbulent, the actual values of these quantities can be expected to depart significantly from the values given here. The purpose of these charts is more to indicate general trends rather than to represent exact results. However, the calculations have been carefully made so that when the assumptions are met the results should be accurate.

### Symbols

$f_p$  = plasma frequency (cycles/sec)

$N_e$  = electron concentration (electrons/meter<sup>3</sup>)

$\nu_c$  = electron collision frequency (collisions/sec)

## CHART DERIVATION

### STAGNATION REGION

The plasma frequency and electron collision frequency for air behind a normal shock (the stagnation region of the blunt body flow field) are shown as functions of ambient density (altitude) and shock velocity in Figure 1. The basic assumption involved here is that the air is in chemical equilibrium. The plasma frequency ( $f_p$ ) was calculated from the electron concentration in the usual manner ( $f_p = 8.977\sqrt{N_e}$ ), where the electron concentration ( $N_e$ ) was taken from Hochstim's<sup>(1)</sup> chart of normal shock properties. The electron collision frequency ( $\nu_c$ ) was then obtained from Musal's<sup>(2)</sup> report for the temperature and density of the shocked air obtained from Hochstim's chart. Since the electron collision frequency for air at temperatures above 8,000° K was not calculated in this report, the electron collision frequency contours in Figure 1 do not extend to the very high velocity regime where such temperatures are encountered. It is in this regime that the ion concentration becomes sufficiently large to become the dominant contributor to the electron collision frequency, at which point the concept of binary collisions between electrons and other particles (upon which the electron collision frequency calculation is based) becomes inadequate to handle the calculation correctly.



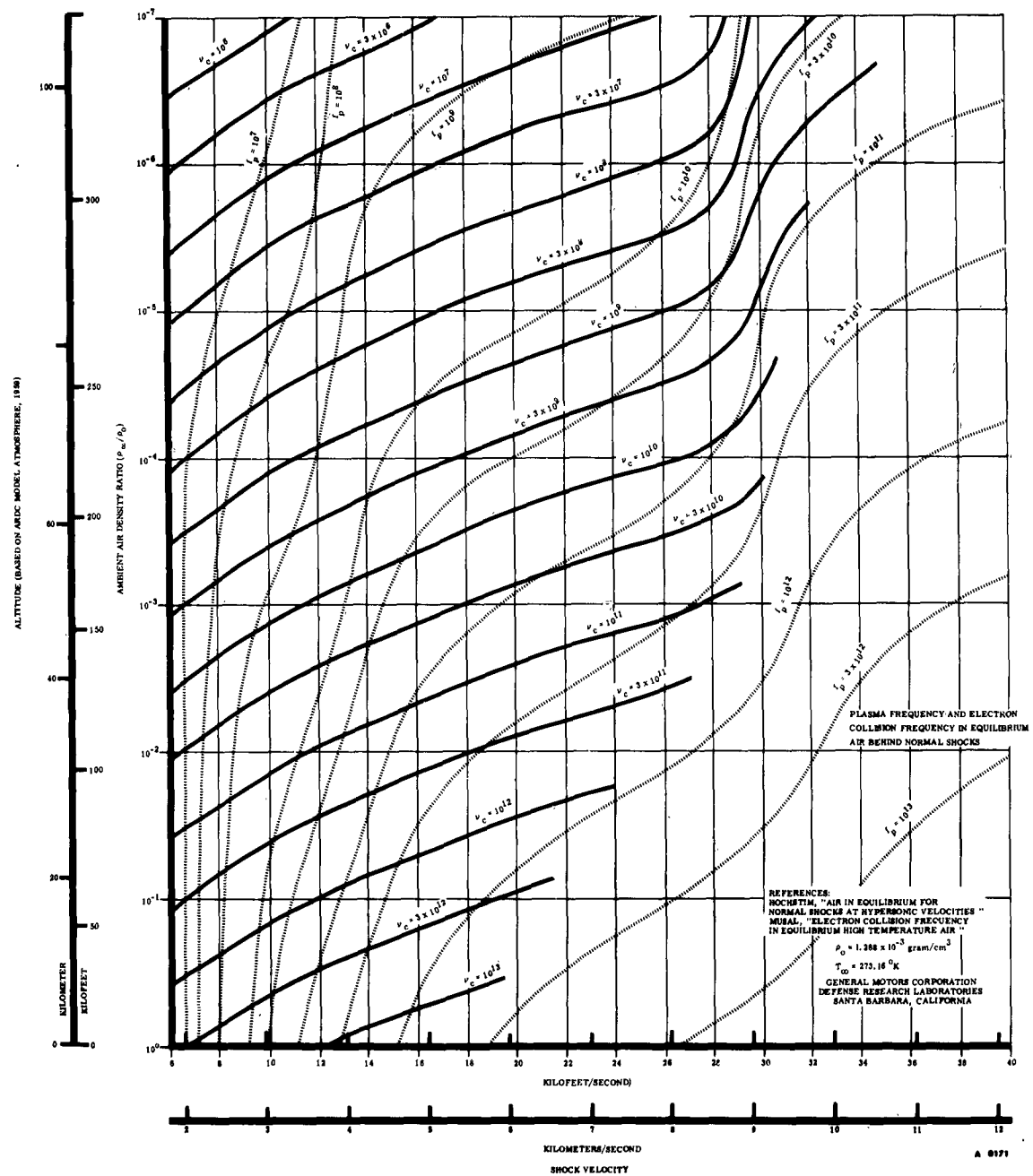


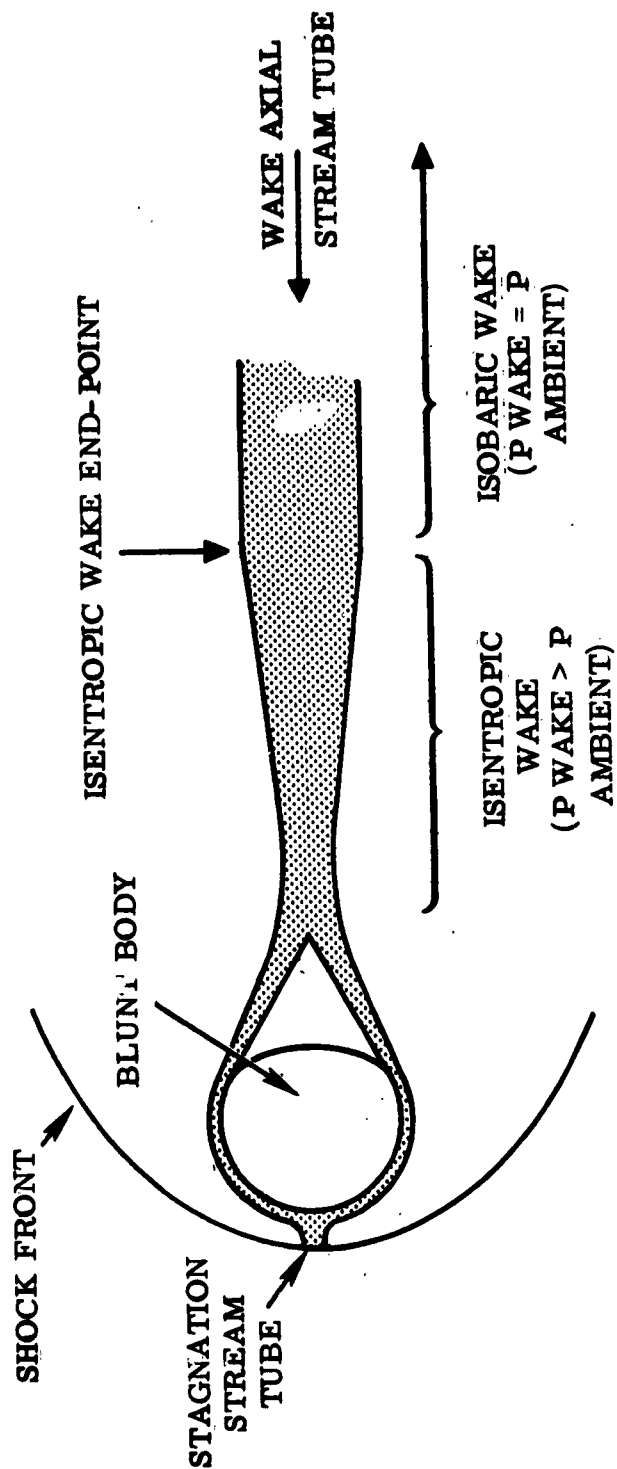
Figure 1 PLASMA FREQUENCY AND ELECTRON COLLISION FREQUENCY IN EQUILIBRIUM AIR BEHIND NORMAL SHOCKS

It may be seen from Figure 1 that the plasma frequency depends most strongly upon the shock velocity, although the ambient density also has a significant influence. The electron collision frequency depends most strongly on the ambient density except at very high velocities.

### ISENTROPIC WAKE

The theoretical study of the flow field behind a hypersonic blunt body (the wake or trail) is extremely complex and still a subject of advanced research. Recently Feldman<sup>(3)</sup> has given an excellent description and analysis of certain aspects of this problem, which will not be repeated except for those points directly related to the results presented here. Following Feldman, the wake behind a hypersonic blunt body is considered to be composed of two regions, first the isentropic (expansion-controlled) wake immediately following the body and behind this the isobaric (conduction or diffusion-controlled) wake, or trail, as shown in Figure 2.

The isentropic wake is assumed to be formed from the hot compressed air in the stagnation region flowing around and behind the body in an equilibrium, laminar, isentropic expanding flow. This expansion stops when the pressure in the wake decreases to the ambient pressure, which marks the end-point of the isentropic wake and the starting point of the isobaric wake. The stream tube from the stagnation point at the front of the body becomes the axial stream tube in the wake when the flow closes behind the body, neglecting the effect of the viscous boundary layer gas which actually flows into the wake core and occupies the axial region of the wake. The air properties



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Figure 2 Wake Nomenclature

are determined along this stream tube from the stagnation point conditions (given by the Hochstim chart) by an isentropic expansion to ambient pressure through the use of the Mollier diagram for equilibrium air <sup>(4)</sup>. The temperature and density thus determined at the isentropic wake end-point are used to find the electron concentration and electron collision frequency from Hochstim and Musal, respectively. The plasma frequency is then calculated as before.

Figure 3 shows the plasma frequency and electron collision frequency on the wake axis at the end-point of the isentropic wake as functions of the ambient density (altitude) and velocity of the body generating the wake. It should be pointed out that because these end-point properties are affected slightly by the ambient temperature, the chart is properly constructed in terms of ambient density and an ambient temperature equal to the reference temperature but not in terms of altitude because of the ambient temperature variation with altitude <sup>(5)</sup>. However, the temperature variations with altitude do not change the end-point values by more than a factor of two under the worst conditions. The direction of change is an increase in both the plasma frequency and the electron collision frequency with an increase in the ambient temperature. The fact that the entire ranges of ambient density and velocity are not covered in this chart is the result of an insufficient range of variables covered in the Mollier diagram. Although the distance behind the body at which the isentropic wake end-point occurs is of interest, it unfortunately cannot be obtained by the simple approach used here.

It may be seen from Figure 3 that the electron collision frequency depends almost solely on the ambient density. At low velocities the plasma frequency depends strongly on velocity; at high velocities

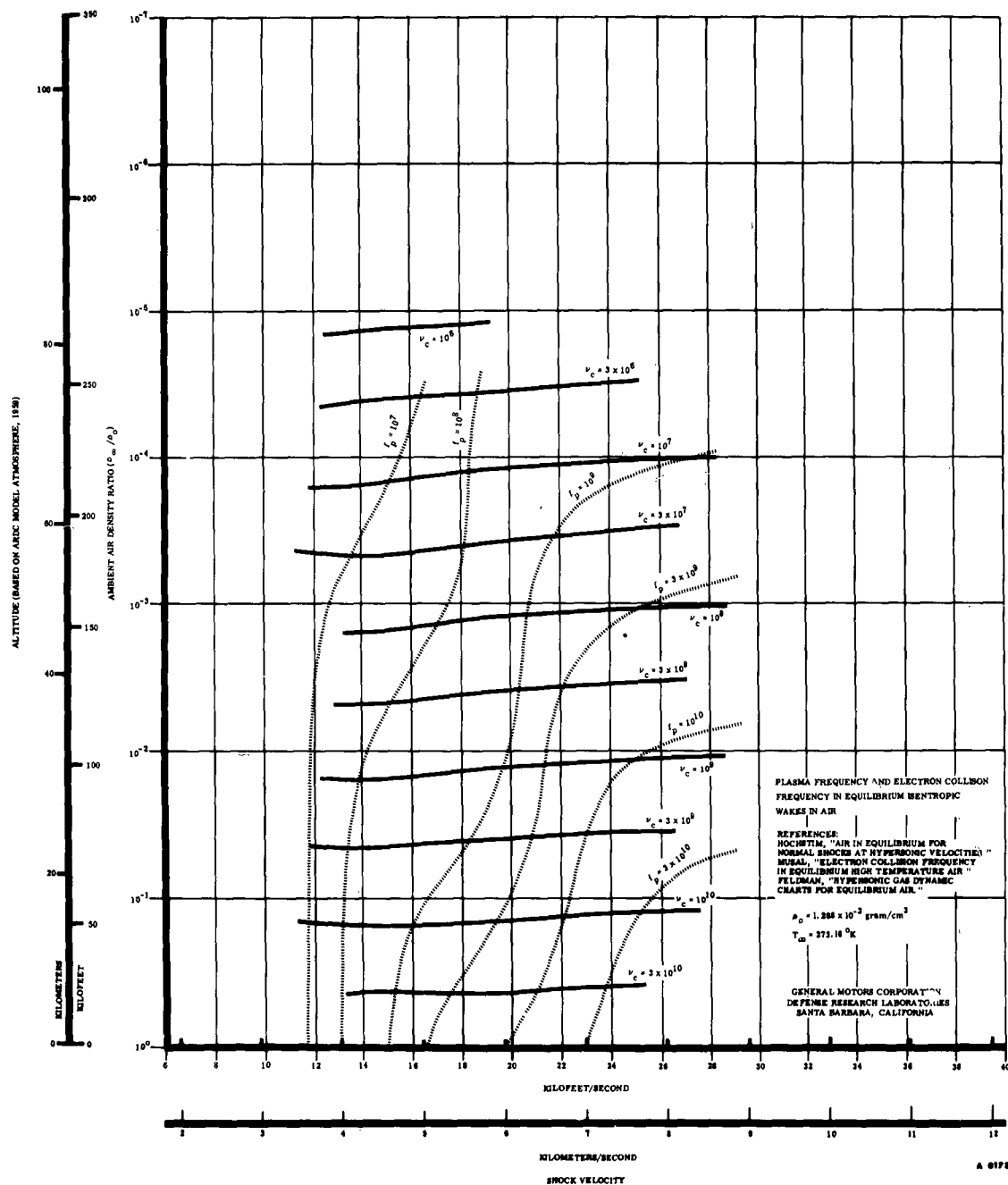


Figure 3 PLASMA FREQUENCY AND ELECTRON COLLISION FREQUENCY IN EQUILIBRIUM ISENTROPIC WAKES IN AIR

there is strong dependence on ambient density. That is, it can be seen that at increasingly high velocities the plasma frequency tends to remain relatively constant.

Some mention of the effects to be expected when the actual flow field conditions deviate from the assumed conditions should be made. Non-equilibrium conditions can affect the plasma frequency considerably, either increasing or decreasing it depending upon the region of the flow field in which the onset of non-equilibrium occurs. Onset of non-equilibrium behind the body would most likely cause the plasma frequency in the wake to remain at higher values than depicted in the chart. Non-equilibrium onset occurring in the stagnation region affects the plasma frequency in the wake in a generally unpredictable manner. Turbulence in the wake will tend to cool the central core, causing the plasma frequency to decrease below the values shown in the chart.

#### CONCLUDING REMARKS

Figure 4, composed from Figures 1 and 3, gives both the plasma frequency in the stagnation region and the isentropic wake end-point in order to show the large range of variation encountered in the flow field from the stagnation point to the isentropic wake end-point. It may be seen that the plasma frequency decreases two or three orders of magnitude from the stagnation point to the end-point of the isentropic wake.

In principle, the results presented here are contained in the original referenced works of Hochstim, Feldman and Musal. However,

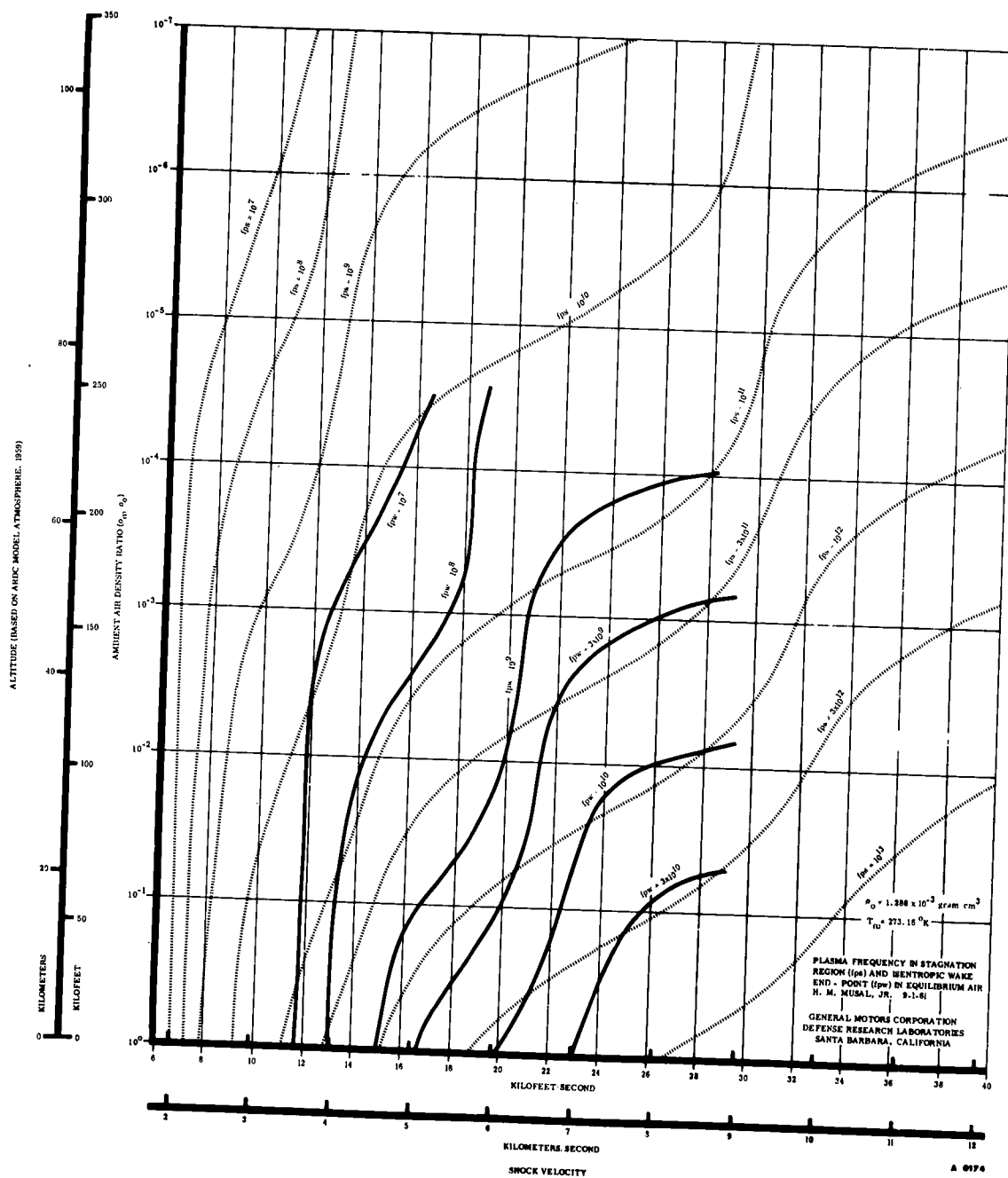


Figure 4 PLASMA FREQUENCY IN STAGNATION REGION (fps) AND  
ISENTROPIC WAKE END-POINT (fpw) IN EQUILIBRIUM  
IN AIR

neither Hochstim nor Feldman (in their primarily aerodynamic studies) give the electron collision frequency in the flow field, an essential parameter for electromagnetic considerations. Feldman (in the wake studies) has given complete and detailed results for a few specific cases, whereas the objective of this study is to cover a broad range of aerodynamic variables without the extensive computational effort required in the detailed analysis. Thus, the wake results presented here are not so comprehensive and detailed as could be obtained using Feldman's approach. However, the extensive range of variables covered here gives a broad survey of the electromagnetic properties of the flow field over a wide range of flight conditions.



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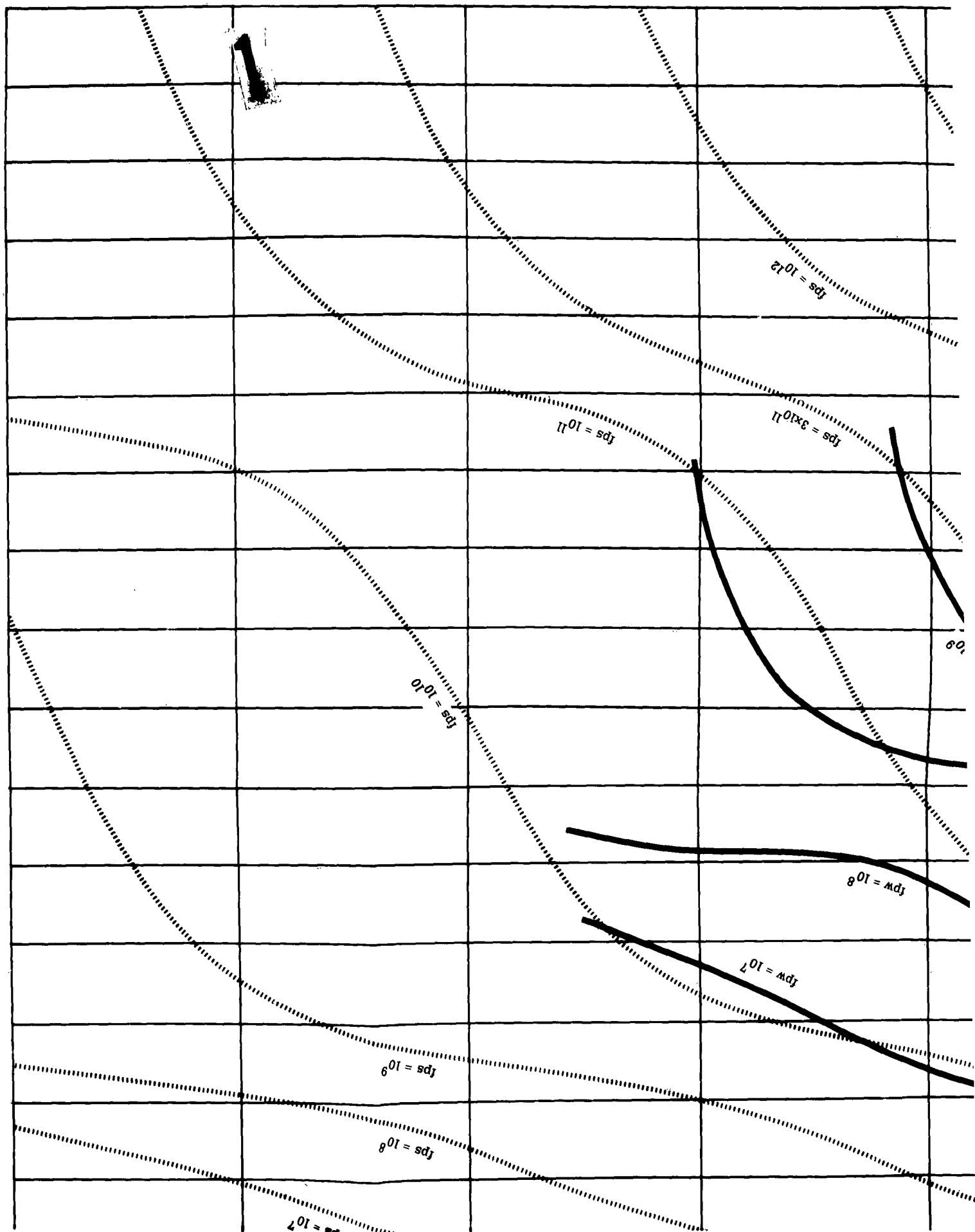
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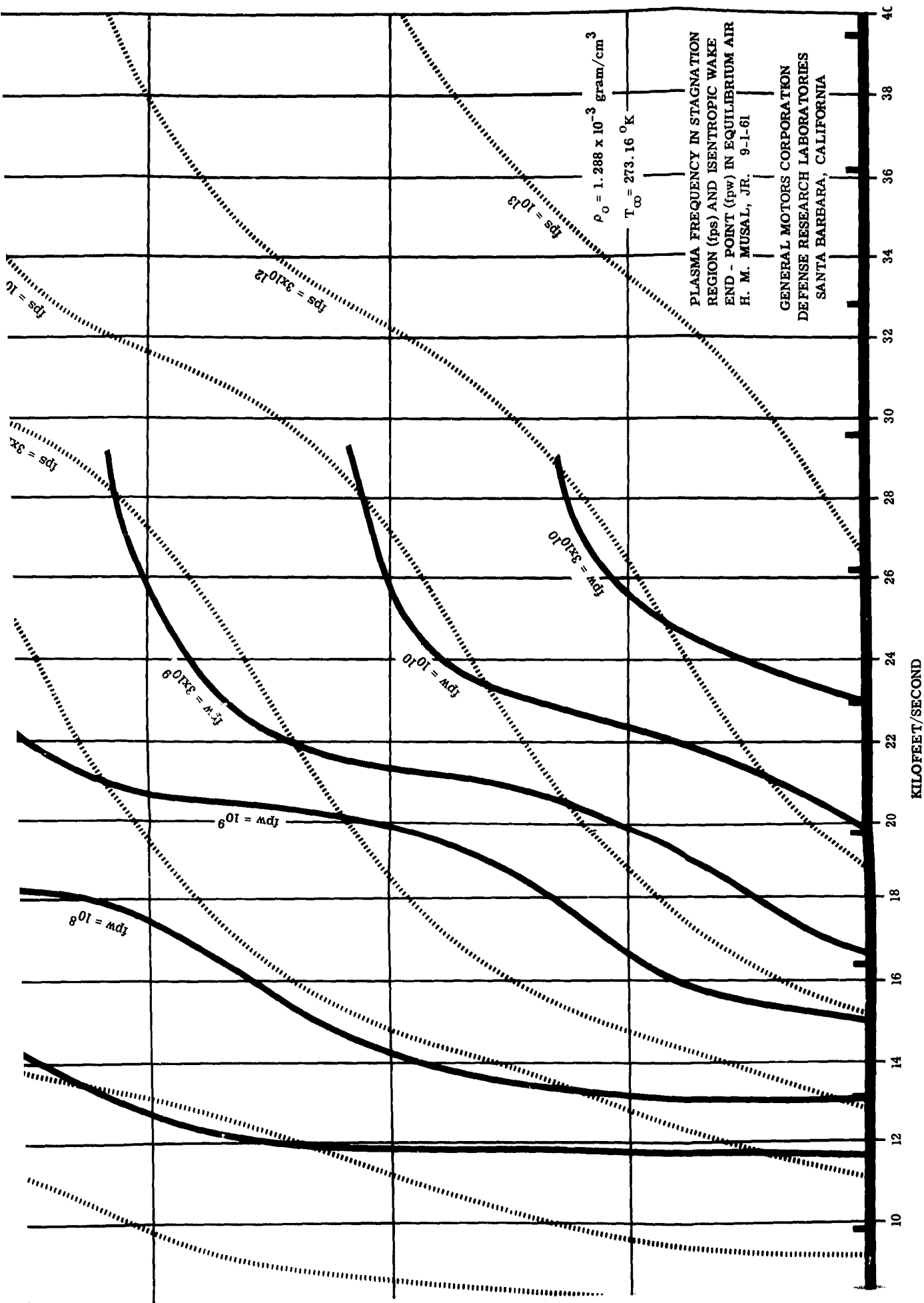
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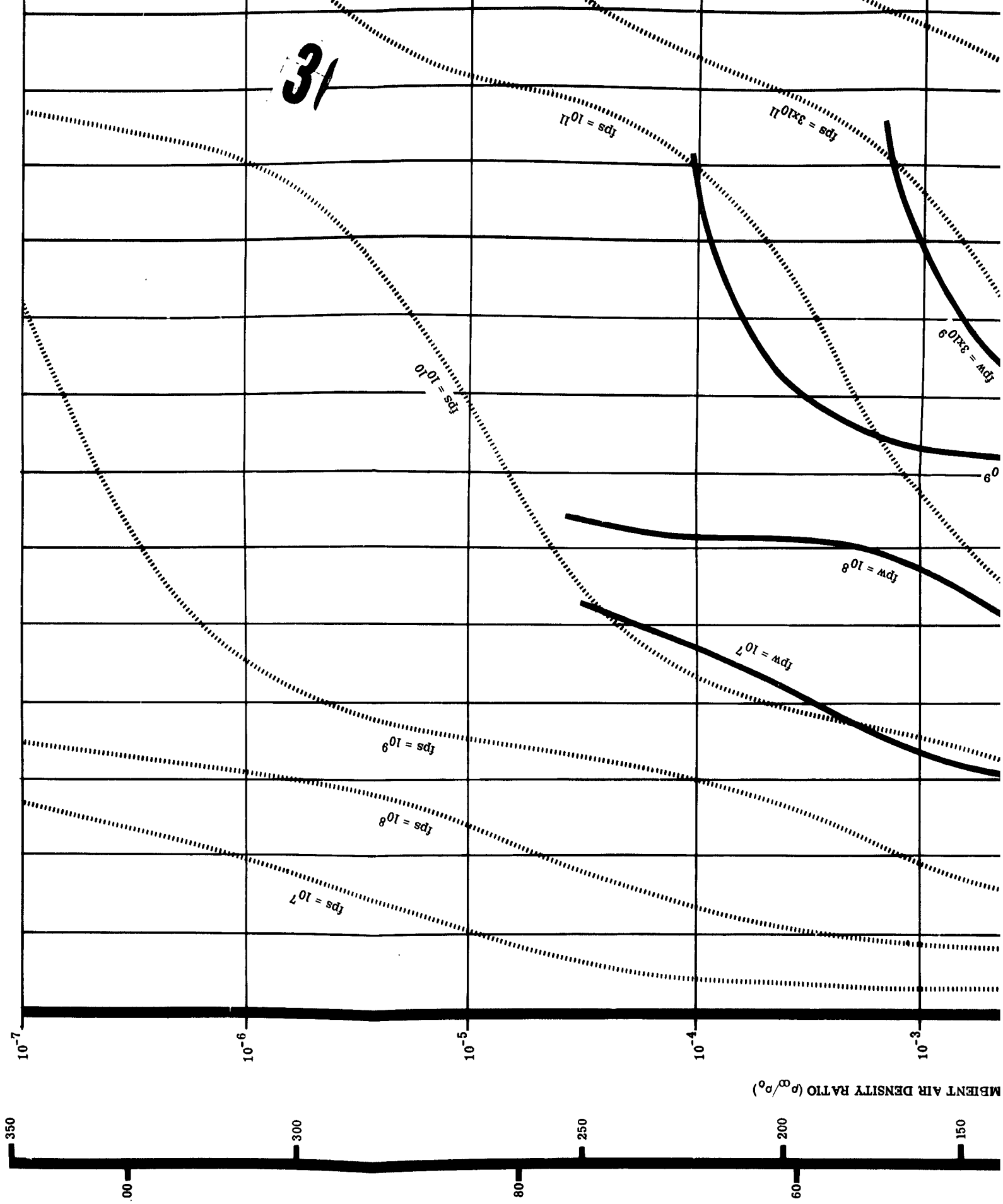
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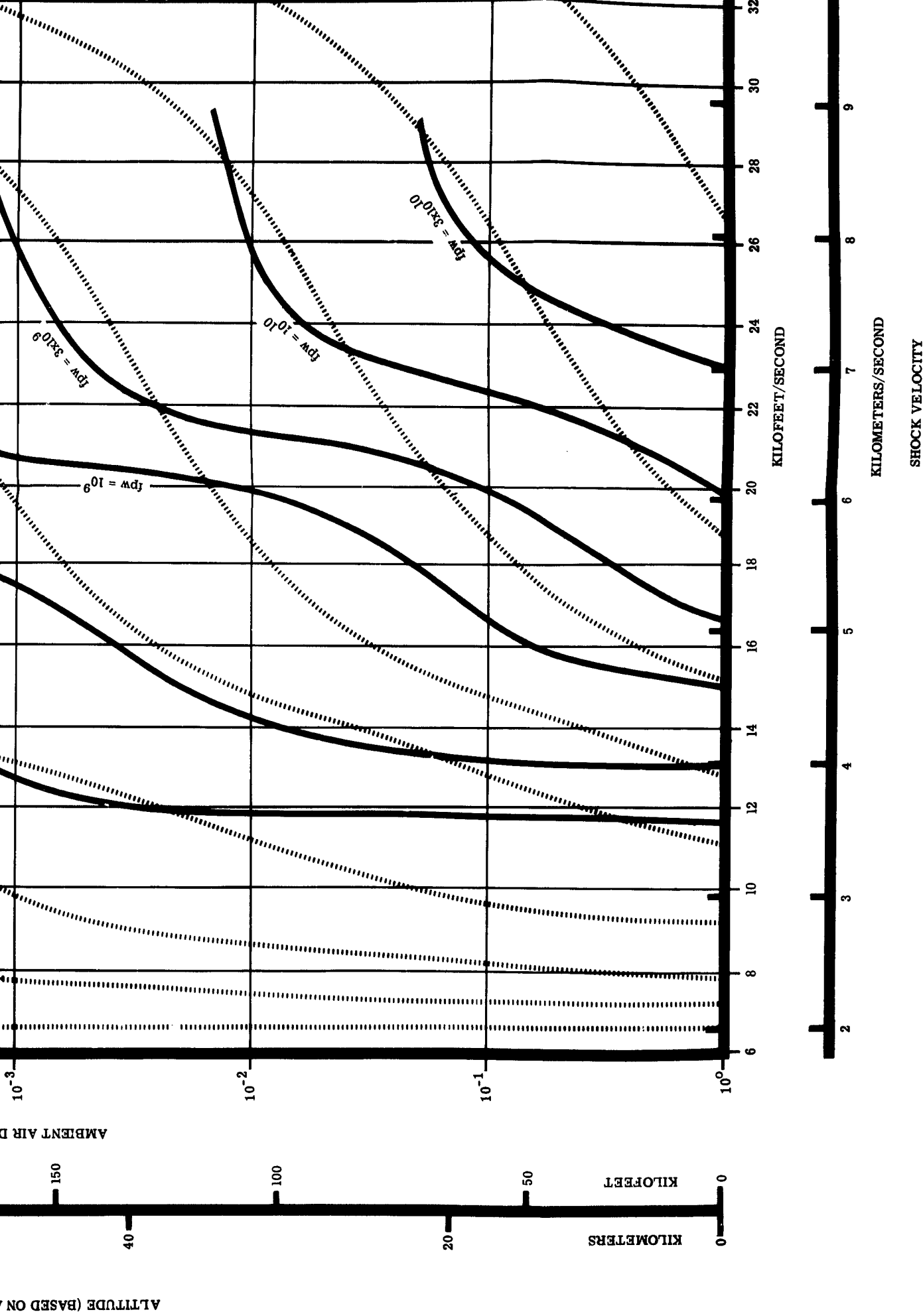




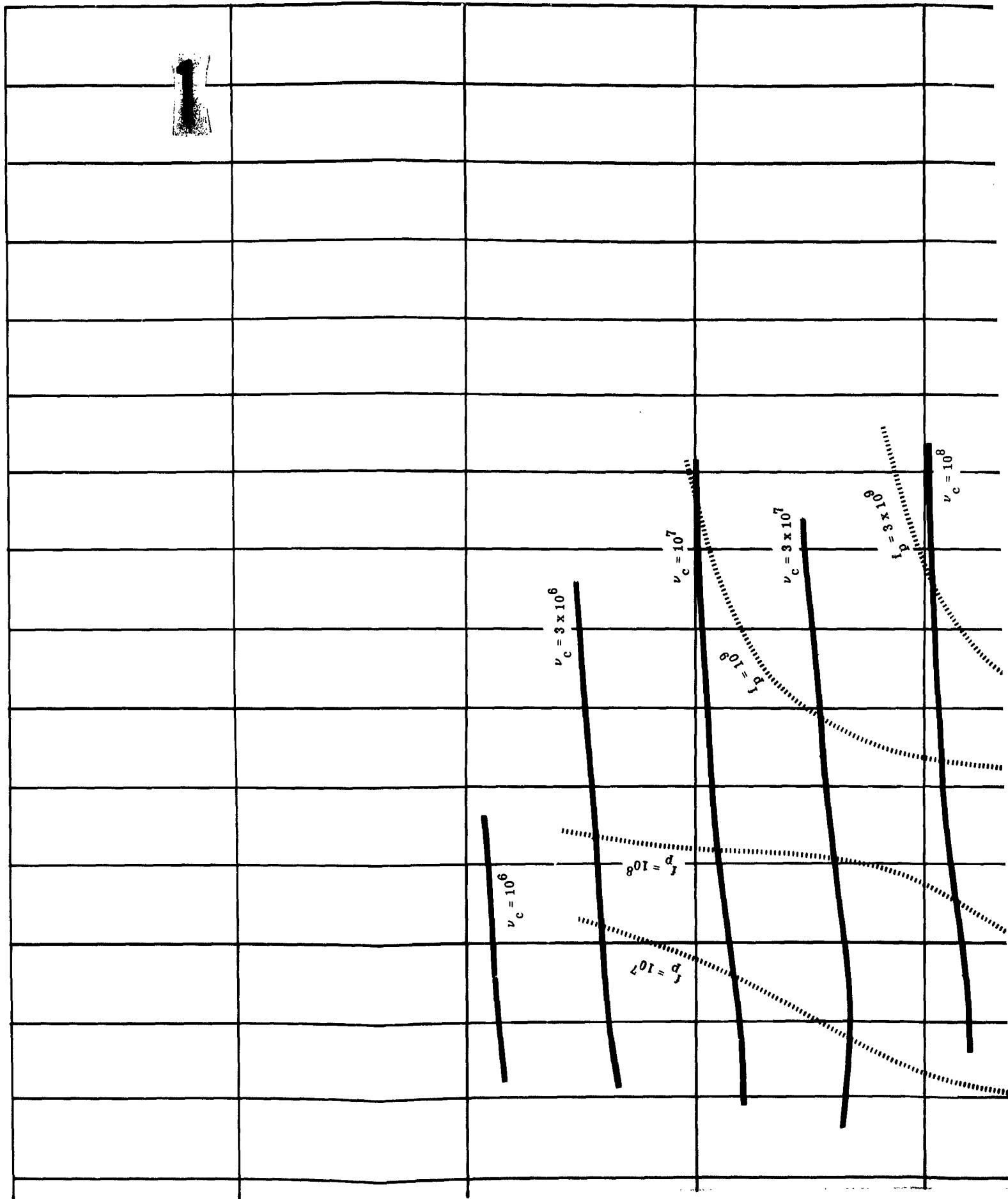
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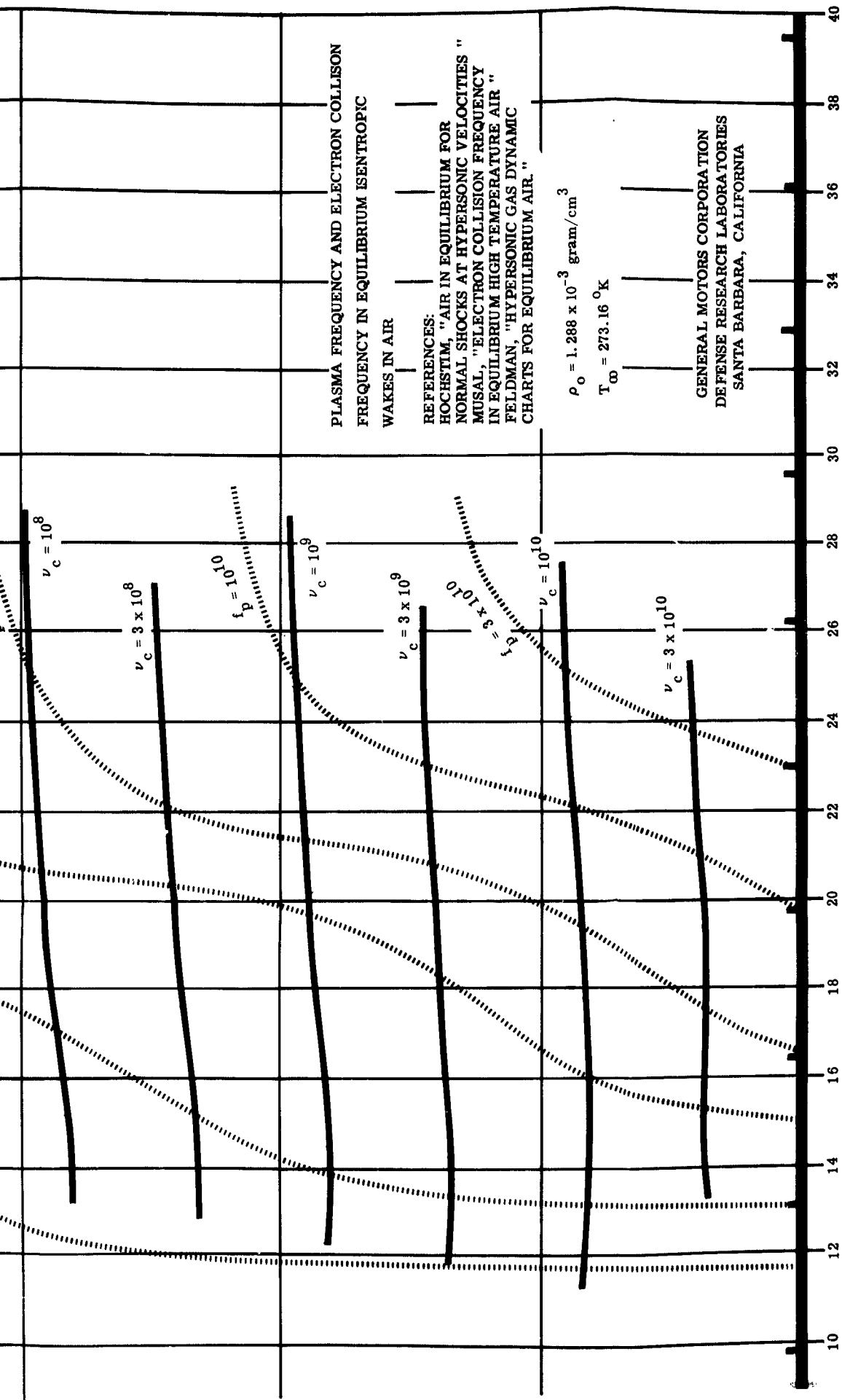












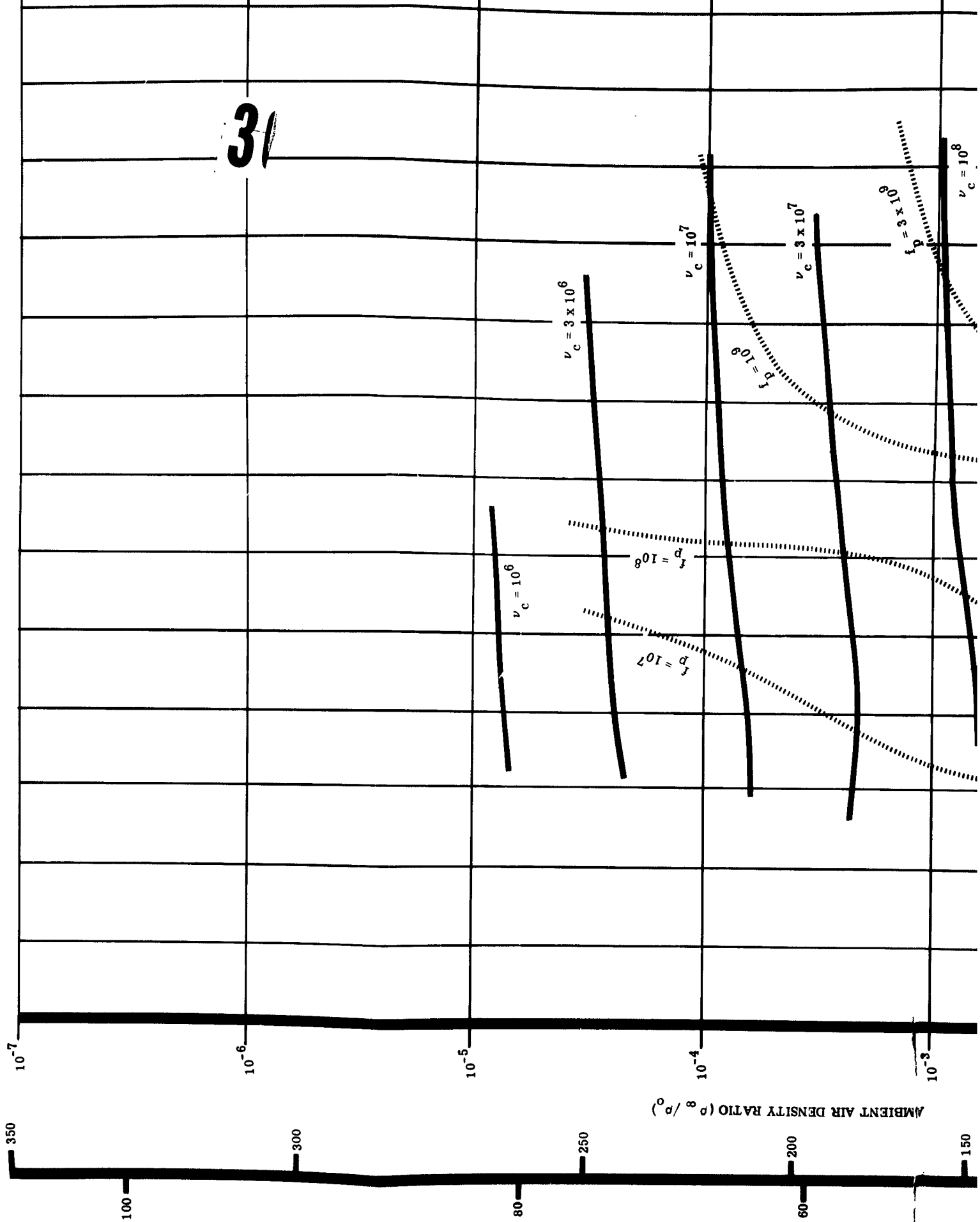
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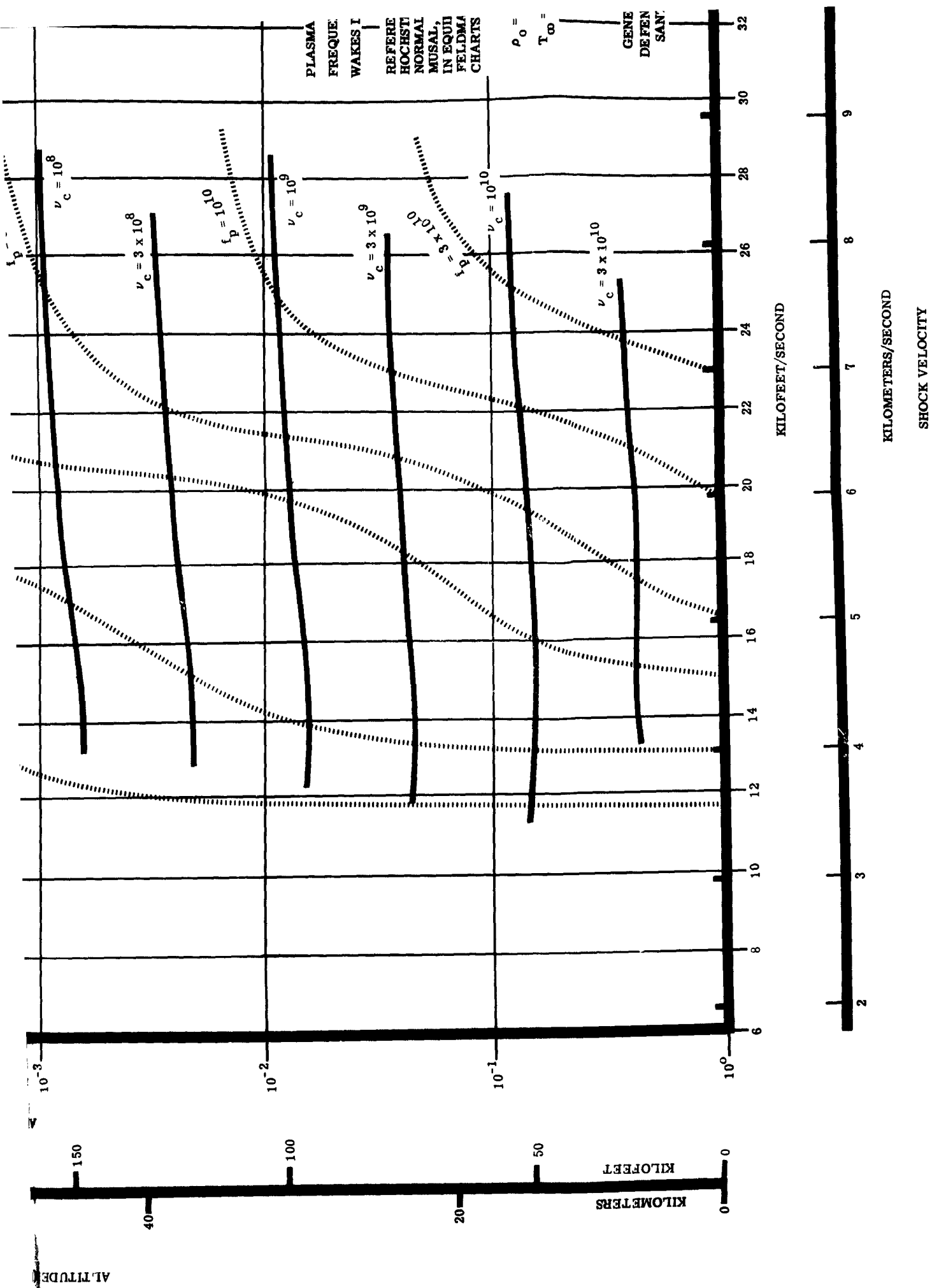
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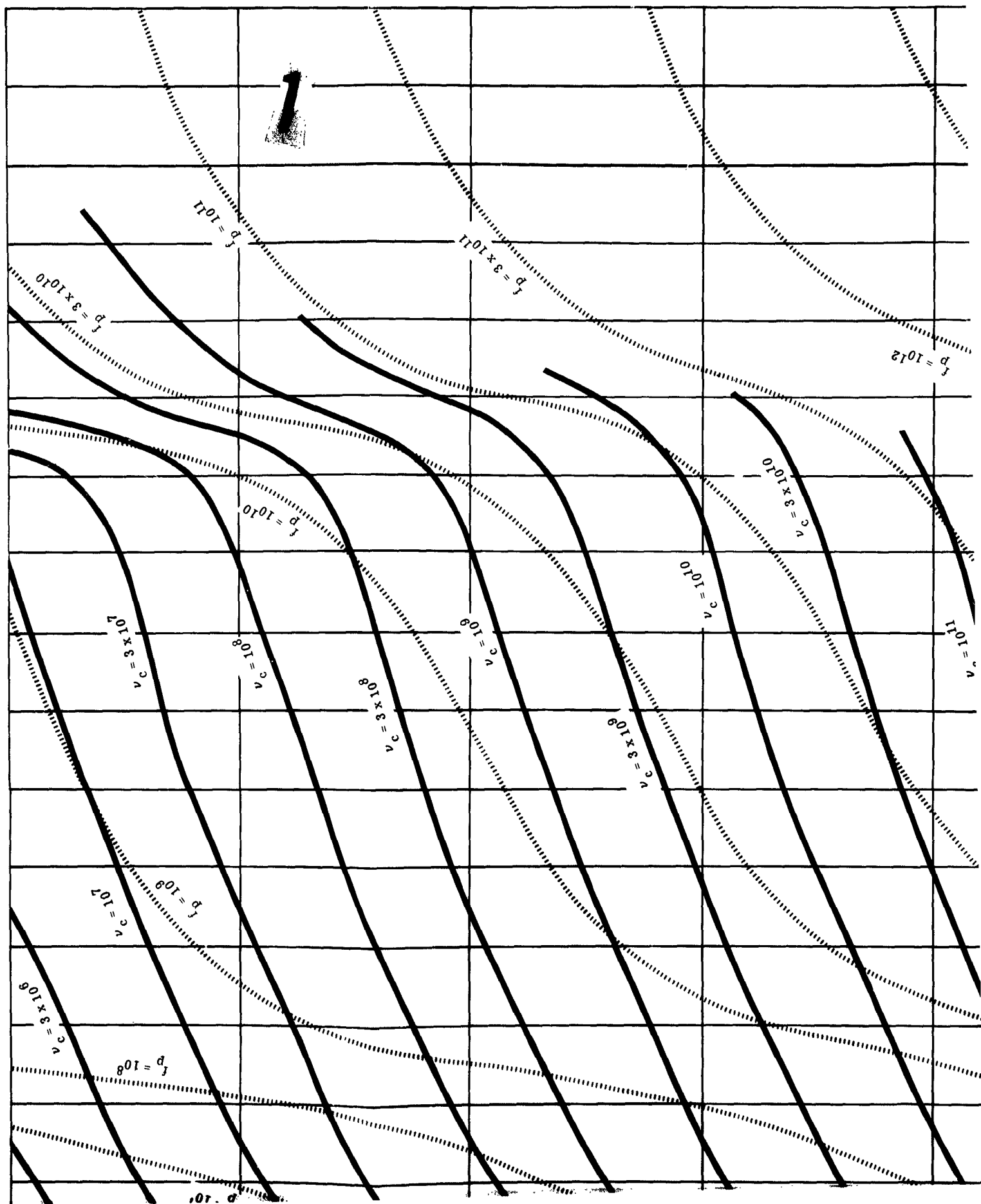
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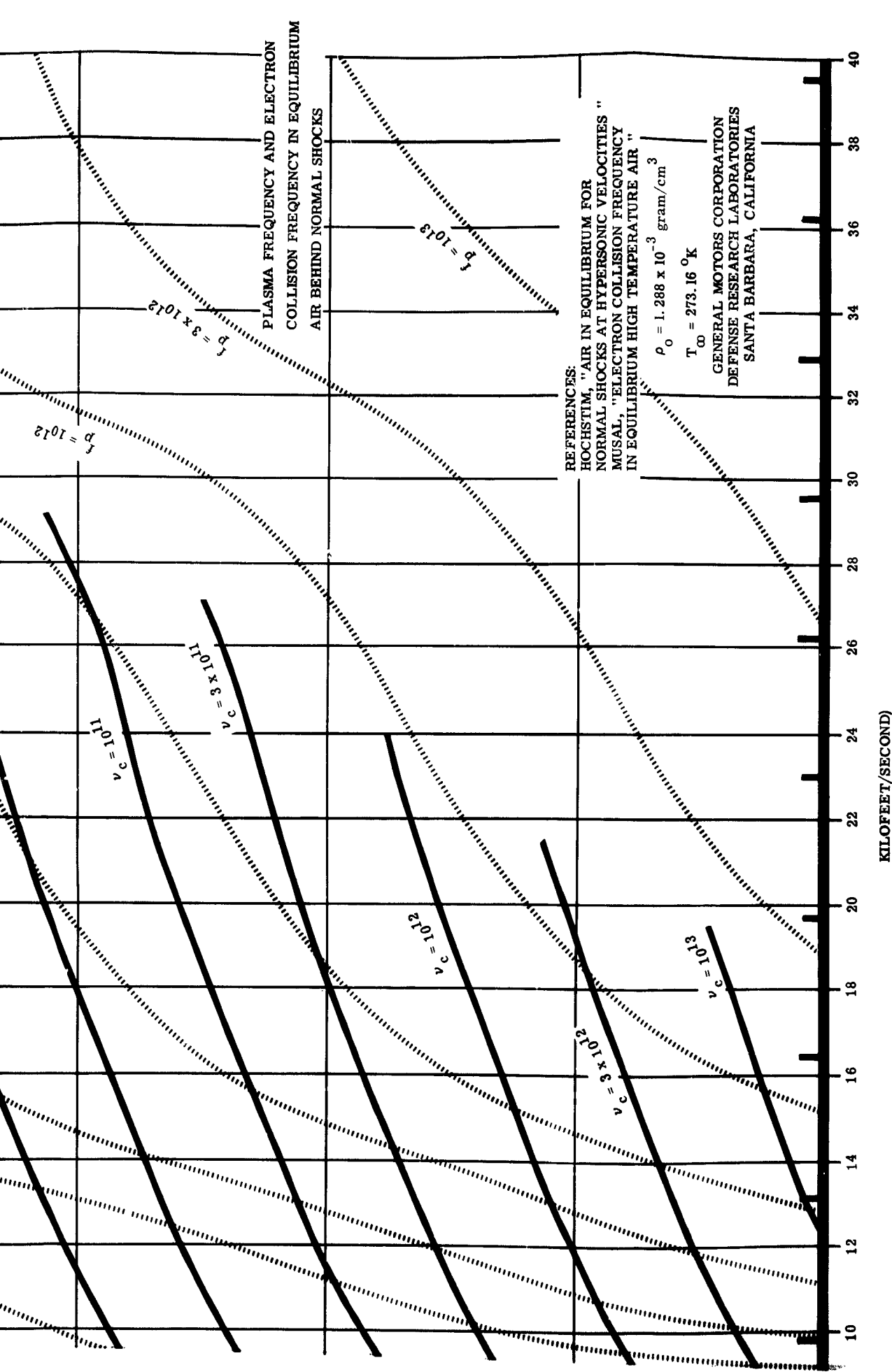
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